Performance improvement of OWC systems by Parametric Resonance.

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ABSTRACT

Peak performance of most OWC systems occurs at resonance with the driving waves. At resonance, oscillations increase linearly in time until damping inhibits further growth. In parametric resonance, oscillations increase exponentially in time, possibly allowing for increasing the performance of OWC systems. This type of resonance occurs when one of the parameters in an oscillator varies periodically. Some ideas are presented to improve the performance of OWC systems using this phenomenon.

Keywords: wave energy, oscillation water column, pump, tuning, parametric resonance.

1. INTRODUCTION

The water column in most OWC systems oscillates under the effect of gravity and the compression of an air chamber, similar to a damped harmonic oscillator. Traditionally, the water column in these systems is excited by the external force exerted by the waves. At best, the natural frequency of oscillation of the system coincides with the driving wave frequency so that the system resonates and the amplitude of the oscillations grows linearly until the damping losses limit growth to a maximum value. Several systems of wave energy conversion have been developed using this phenomenon (1, 2, 3, 4, 5, 6, 7).

In an oscillator, the normal modes of oscillation are a function of several parameters, such as the mass and the spring constant. When one of these parameters changes periodically, a small perturbation can grow exponentially in time. This phenomenon is known as 'parametric resonance' (8). In classical mechanics, it is known that when this type of resonance occurs, energy is transferred with much greater efficiency than in ordinary resonance. The increased oscillations in a child's swing, in the absence of external forces, is a simple example of parametric resonance. In this case, the parameter being rhythmically changed is the length of the rope, as the child rises and lowers his centre of mass. The work exerted by the child in doing so is transformed very efficiently into mechanical oscillations. In this paper, some basic concepts of parametric resonance are explored as a possible way of improving the performance of OWC systems.

2. BASIC MATHEMATICAL MODEL OF PARAMETRIC RESONANCE

A basic generic OWC system can be seen in figure 1. Applying the Bernoulli equation to the streamline from points 1 to 2 in figure 1, the basic equation of the system can be obtained (9). Non-linear losses due to friction, vortex formation and radiation damping (third term) have been included in the same way as (7).

$$(X + L(1+\varepsilon))\ddot{X} + \frac{\dot{X}^2}{2} + \left(\frac{K}{2} + \frac{L}{D}L + C_r\right)\dot{X}\left|\dot{X}\right|$$

$$+\frac{(P_A-\rho gH)}{\rho}\left[\left(1-\frac{AX}{V_o}\right)^{-\gamma}-1\right]+gX=0 \qquad 1$$

where

L, A & D	Resonant duct length, area and diameter.
P_A	Atmospheric pressure (kg m^{-1} s^{-2})
ρ	Sea water density (kg m^{-3})
$\gamma = C_p/C_v$	Air compressibility = 1.4
Vo	Compression chamber volume (m^3)
H	Height of compression chamber equilibrium position above receiving body of water (m)
ε	Fractional added length due to edge effects at the duct mouth
g	Gravitational acceleration
F	Friction loss coefficient for oscillating flow in pipes.
K	Vortex formation energy loss coefficient (see 9)
C_r	Radiation damping coefficient (see 9).

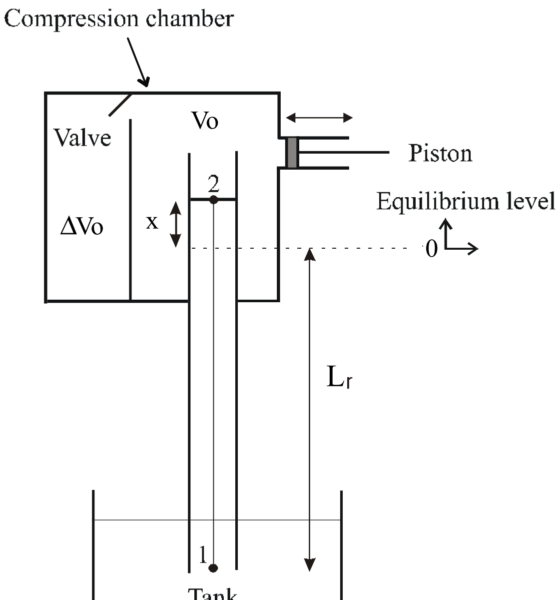

Figure 1. Schematic diagram of the oscillating water column.

The first, second and fifth terms in eqn. 1 come directly from Bernoulli's equation while the air compression term (4th), was obtained assuming adiabatic conditions.

For small oscillations, the non-linear terms are negligible and eqn. 1 can be rewritten as

$$\ddot{X}+\omega_0^2 X=0 \qquad 2$$

where

$$\omega_0^2=\gamma\frac{(P_A-\rho gH)A}{\rho V_0 L(1+\varepsilon)}+\frac{g}{L(1+\varepsilon)} \qquad 3$$

is the square of the natural frequency of oscillation of the system. If one of the parameters in eqn. 3 changes periodically in time, to a first approximation, the frequency of oscillation can be written as

$$\omega^2(t)=\omega_0^2+\beta Cos\,Wt \qquad 4$$

Equation 2 with the frequency defined by 4 is known as Mathieu's equation, which is a typical system in which parametric resonance can occur (10). This type of equation can be solved using the Floquet theory (11) where the solution is the product of an exponential function and a periodic function. When the argument of the exponential function is real, the solution grows exponentially in time. Otherwise, the solution is the product of two bound periodical functions. We now explore in which conditions the argument in our case can be real, that is, the values of ω_0/W and β where parametric resonance can occur.

Following a standard asymptotic method, such as described in (12), it can be shown that the solution to Mathieu's equation has the bifurcation diagram shown in figure 2.

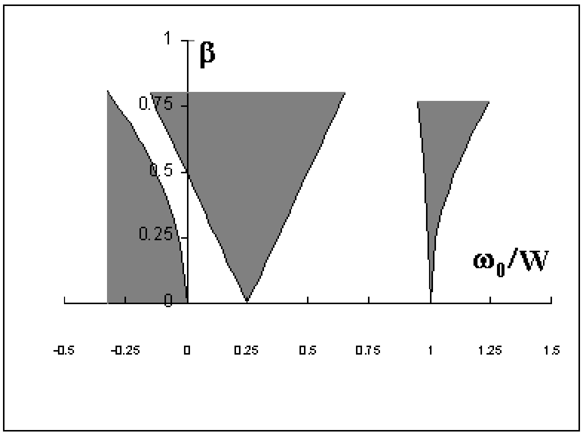

Figure 2. The shaded regions represent the unstable solutions of Mathieu's equation.

The darkened areas in this figure cover the values of ω_0/W and β where the solutions to Mathieu's equation grow exponentially in time. It can be seen, in contrast to normal resonance which occurs at unique values of the driving frequency, that parametric resonance can occur in several regions which grow from critical values of $\omega_0/W = n^2/4$ (n = 0, 1, 2, …) at $\beta = 0$. The main parametric resonance occurs when $W = 2\ \omega_0$ ($\omega_0/W = 0.25$).

3. NUMERICAL EXPERIMENTS

Theoretically, any of the parameters in eqn. 3 can be made to vary periodically to induce parametric resonance in the system shown in figure 1. Nevertheless, we can only envision practical ways to modify, for example, the volume of air in the compression chamber. In order to visualise the results that can be obtained by introducing a variation in this parameter, we resort to the numerical model of a wave-driven seawater pump described in (7). The pump is described by two equations very similar to eqn. 1, coupled by the air compression chamber term.

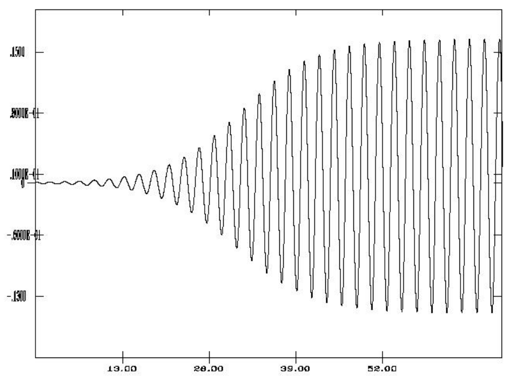

Figure 3. Amplitude of oscillation vs. time of the numerical model described in reference (7).

In the numerical experiments, the volume of air in the compression chamber was made to vary 30 % with double the frequency of the normal mode of oscillation for that particular volume, with no external wave excitation. In figure 3, the oscillations of the water column in the seawater pump can be seen initially to grow exponentially with time, starting from a small perturbation. After a certain point, growth increasingly diminishes until the oscillations reach a constant amplitude. Physically we cannot expect an indefinite exponential growth in time. After a certain point, the non-linear terms in the full equations will increase in size, eventually bounding the attainable growth.

4. NONLINEAR PARAMETRIC OSCILLATIONS

In previous works we have studied the effect of the non-linear terms in the seawater pump equations (7). We found that vortex formation and radiation damping are the main losses in our system. These non-linear losses correspond to the third term in eqn. 1. In order to study the effect of such terms on Mathieu's equation we include the following in eqn. 2.

$$\beta\eta\dot{X}\left|\dot{X}\right|$$

The standard method to solve this type of non-linear equation is the multiple scales asymptotic method as described in (12). Our goal is to find the maximum amplitude of oscillation in this non-linear equation in the presence of the above losses. The maximum attainable oscillation amplitude is given by

$$A_v = \frac{6\beta}{\omega_0^{\,2}\eta}\sqrt{1-4\omega_1^{\,4}}\ .$$

Here ω_1 is the shift from the natural frequency of oscillation ω_0, due to non-linear effects. It should be noted that once this amplitude is reached, it no longer varies in time.

The main characteristics of the solutions are the following:

a) The attainable amplitude is proportional to the inverse of the coefficient η, which is due to vortex and radiation damping losses. This is in contrast with the dependence on $1/\eta^{1/2}$ of the maximum attainable amplitude in normal resonance.

b) The solutions grow exponentially at first until they reach the maximum attainable amplitude. The solution then keeps this amplitude despite small variations in ω_0. This is in contrast with normal resonance that exhibits a modulatory behaviour when the driving frequency does not coincide exactly with the system natural frequency of oscillation.

c) Small changes in the frequency of the parametric excitation, results in negligible reductions in the oscillation amplitude. To a first approximation, the amplitude depends on the de-tuning term ω_1 as $A_v \sim 1\text{-}2\omega_1^4$. In ordinary resonance, the amplitude depends, more significantly, on the de-tuning term as $A_v \sim 1\text{-}\omega_1^2$.

5. A POSSIBLE PRACTICAL APPLICATION.

A simple implementation of parametric resonance to an OWC can be described using figure 1. Here, the main compression chamber is connected to an adjacent volume of air by means of a valve. When this valve is open, the total air volume increases, softening the air spring restoring force. In this way, opening and closing the air valve modifies the air spring constant, therefore changing one of the parameters of the system. The piston shown in figure 1 can be used to modify the equilibrium level of the water column surface.

In figure 4, a full cycle of oscillation is shown as an aid to describe one way in which parametric resonance could be induced. This cycle is divided into four sections as follows. In the first quarter, that is, when the water level descends from (I) before reaching the equilibrium level at (II), the air valve in figure 1 is closed shut so that the spring is harder, forcing the water column down faster than if the valve were open. Nevertheless, closing the valve at the highest point (I), raises the equilibrium level of the water column surface to point A, above the reference level at point B. In order to take advantage of the increased potential energy of the hardened spring, we must restore the equilibrium level during this part of the cycle, to reference point B, by pushing the piston (P) in figure 1, an appropriate displacement. By doing this, the kinetic energy of the system, when the water level reaches (II), will be greater than if the valve had been left open, at the expense of the work performed by the piston.

In the next quarter cycle, that is, from (II) to (III), the valve is now kept open to soften the air spring, so that the kinetic energy at point II will transform into a greater displacement at point III. Opening the valve at point II, however, lowers the equilibrium level to point C so that the piston must be extracted to restore this point to B.

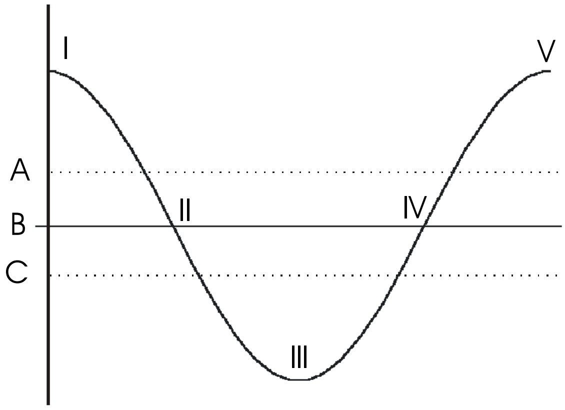

Figure 4. Water level of the oscillating water column vs. time. Point B represents the reference level.

During the remaining two quarter-cycles, that is from (III) to (IV) and from (IV) to (V), the air valve and the piston are operated in a similar way. The aim is to harden the air spring when speed of the water column is increasing and soften it when speed is decreasing. External work is done by pushing and pulling the piston, in order to keep the average equilibrium level fixed.

When the spring is soft, the natural frequency of oscillation can be defined as ω^- which is smaller than ω_0 by a certain $\Delta\omega$, which depends on the relative size of the two air chambers connected by the valve. In a similar way, when the spring is hard, the corresponding frequency is ω^+. When the process described in the previous two paragraphs is followed, it can be shown that the oscillation amplitude will increase by $(\omega^+/\omega^-)^2 > 1$ from cycle to cycle. That is, the increase is proportional to the amplitude of the previous cycle so that there is an increase, in effect, exponential in time. This is in contrast with normal resonance where the amplitude increases a constant amount from cycle to cycle, which is linear in time.

6. DISCUSSION AND CONCLUSIONS

It has been shown that parametric resonance may be a novel way to induce greater oscillations in an OWC system. In classical mechanics, energy is transferred more efficiently by parametric resonance than when an external excitation occurs. The exponential growth of the amplitude provides a very fast response for any initial perturbation and the maximum attainable amplitude is greater than in common resonance.

With the type of energy losses found in typical OWC's, in parametric resonance, once the system reaches the maximum amplitude, the oscillation does not modulate even if it is not exactly in tune, in contrast to normal resonance where modulation always occurs when out of tuning. In addition to this, the amplitude of the oscillation in parametric resonance is less sensitive to variations in the resonant frequency.

An effective way of using wave energy to perform the work of the piston is yet to be found. Nevertheless, parametric resonance opens the way to new designs and different uses of OWC's. In the case of fluid pumps, not necessarily of seawater, this phenomenon could be used for bi-directional flow.

7. ACKNOWLEDGEMENTS

We wish to thank the Instituto de Ciencias del Mar y Limnología (Project "Energía de Oleaje" No. 139), from the National University of Mexico (UNAM). Support was also provided by CONACyT through project 'Matemáticas no-lineales en la Física y en la Ingeniería, No 625427-E.

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